GROSS CUBIC-VOLUME EQUATIONS AND TABLES, OUTSIDE BARK, FOR PINYON AND JUNIPER TREES IN NORTHERN NEW MEXICO

Gary W. Clendenen

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RESEARCH SUMMARY

Presents cubic-volume equations and tables for estimating gross cubic volume outside bark of individual pinyon and juniper trees in northern New Mexico; also shows procedures used in building mathematical models.

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INTRODUCTION

Pinyon and juniper trees are commonly small and scrubby, have rapid taper and a bushy appearance. These characteristics, which are different from those of tree species conventionally managed for production of sawlogs, give both the private landowner and the public forester some unusual problems in estimating wood volume. Few volume tables have ever been published, and volume equations are virtually nonexistent. Howell and Lexen published some tables in the late 1930's and early 1940's (Howell and Lexen 1939; Howell 1940 and 1941) and Moessner later published an aerial photo volume table (1962). All these tables were published within one cover by Barger and Ffolliot (1972).

DATA COLLECTION

The lack of adequate equations or tables for estimating cubic volumes of pinyon and juniper plus the interest of personnel on the Carson National Forest in New Mexico in assessing the current situation in their pinyon and juniper resources resulted in plans for a cooperative study. Participants were the Intermountain Forest and Range Experiment Station's Renewable Resources Evaluation work unit, the Carson National Forest, and the Southwestern Region of the Forest Service. The data used to develop the cubic volume equations presented here were collected on the Carson National Forest as part of this cooperative study. Independent variables collected and used in the data analysis were:

1. Species
2. Basal diameter outside bark--diameter at ground line
3. Total tree height
4. Number of 3-in (7.62 cm) and larger forks anywhere on the stem where both stems coming out of the fork were 3 in or larger diameter outside the bark
5. Number of stems 3-in (7.62 cm) diameter and larger outside the bark and originating within the first 12 in (30.43 cm) above ground line.

The two independent variables measured for use in determining cubic-foot volume of each sample tree were: (1) the number of segments in each tree by 2-ft (0.61 m) length classes, and (2) 2-in (5.08 cm) midpoint diameter classes (2 to 8 ft, or 0.61 to 2.44 m). The midpoint diameter classes were even 2-in (5.08 cm) classes; 2-in (5.08 cm) diameters were the smallest. If any segment (except for the 2-ft length class) had more than 2 in of taper, it was divided into two or more segments such that maximum taper within a segment could not exceed 2 in (5.08 cm).

Of the several species of pinyons and junipers native to the Rocky Mountain West, only three were encountered in this study: pinyon (*Pinus edulis* Engelm.) and Rocky Mountain and Utah junipers (*Juniperus scopulorum* Sarg. and *J. osteosperma* [Torr.] Little). Of the total of 2,318 trees measured, 1,559 were pinyons and 759 were junipers. The trees were selected from 96 field plots randomly scattered about the Carson National Forest.

DATA ANALYSIS

Using diameter and length class, the cubic volume was calculated for each segment in each sample tree assuming the segment was a cylinder having diameter equal to the segment midpoint diameter and length equal to the segment length class (Huber's formula).

The segment cubic volumes were then summarized by diameter class to provide estimates of cubic volume for each tree by six different top diameters--1 in through 11+ in (2.54 cm-27.94 cm) by 2-in (approximately 5 cm) odd increments. Cubic volume, so calculated, was used as the dependent variable for regression analyses.

A combinatorial screening approach (Grosenbaugh 1967) was used initially to determine which combination of pertinent transforms of all independent variables gave the best estimate of cubic volume. The results indicated basal diameter and total height to be the best predictors of cubic volume for pinyon. Basal diameter, height, and number of forks provided the "best" correlation with juniper volume. Substituting number of stems for number of forks gave a slightly smaller correlation; however, number of stems was selected for the final juniper volume model because number of stems is more easily and accurately determined in the field.

After these independent variables were selected, the data were summarized by species for each minimum top diameter limit. For pinyon, mean cubic volume was computed for each 2-in (approximately 5 cm) basal diameter class and each 5-ft (1.5 m) height class. For juniper, mean cubic volume was computed for each 2-in (5 cm) basal diameter class, 5-ft (1.5 m) height class, and number-of-stems class. These mean cubic volumes were then used to develop a refined volume model that was finally fitted to all individual tree volumes by linear least squares techniques.

The following constraints were applied to the volume models:

1. Volume should increase over height and must pass through zero cubic volume when height equals zero.

2. Volume should increase over basal diameter and must pass through zero cubic volume when basal diameter equals minimum top diameter limit.

3. Volume must approach zero cubic volume as number of stems increases.

4. Volume must decrease with increasing minimum top diameter limit.

5. The minimum top diameter components must not cross.

6. The number of stem components of model must not cross.

An initial plotting of the data indicated volume increased linearly over height and was concave upward over basal diameter.

The first constraint is satisfied by a model of the form $\hat{Y} = bH$ where b is expressed as a function of basal diameter, number of stems, and minimum top diameter limit. Minimum top diameter limits of 9 in (22.86 cm) and 11 in (27.94 cm) were dropped from the analysis because inadequate sample sizes resulted in inconsistent trends. The final model was then developed using minimum top diameter limits of 1, 3, 5, and 7 in (2.54, 7.62, 12.70, and 17.78 cm).

The first step in the model building process was to fit a weighted linear equation of the basic model form through the origin with height as the independent variable and cubic volume as the dependent variable. This was done for each species, 2-in (5 cm) basal diameter class, minimum top diameter limit, and in the case of juniper, each number-of-stem class up to six. Beyond six stems, the data base was insufficient to determine consistent trends. The weights used were the number of observations contributing to each mean volume.

Next, the resulting slopes (*b* coefficients) were plotted over basal diameter minus minimum top diameter limit. Basal diameter minus minimum top diameter limit was used because of the second constraint mentioned previously. Matchacurve techniques (Jensen 1964, Jensen and Homeyer 1970, 1971, Jensen 1973, 1976) were used to describe the trends of the slopes in relation to basal diameter minus minimum top diameter for each minimum top diameter class. For pinyon, the power function of the form

$$\hat{Y} = aX^{n}$$

was a satisfactory descriptor with a and n to be a function of minimum top diameter alone. For juniper, a single power function did not provide enough upward curvature for the larger basal diameters; therefore, a multiple component model having the following form was used:

$$\hat{Y} = a_1 X^{n_1} + a_2 X^{n_2}$$

where the $a_2X^{n_2}$ component becomes essentially zero for small diameters.

As in step 1, each model component was fitted through the origin by weighted (number of observations) least squares techniques.

For juniper, the slopes were also a function of number of stems. The number-of-stems effect was allowed to asymptote toward zero at 20 stems.

Appropriate power functions for the minimum top diameter limit effect were then determined. The same procedure as used in step 2 was employed, except this time the independent variable was minimum top diameter limit.

The components were combined for each model and the latter were each fitted to their respective data set by least squares techniques. The simple linear model, $\hat{Y} = b\ (model)$, was used, forcing the model through the origin. The resulting coefficients were very close to 1, indicating that the models as developed fitted the overall data trends very well.

Next the residuals were plotted over estimated volume. The plotted residuals showed increasing residuals with increasing estimated volume, indicating a need to weight the estimate of b in the final models. Draper and Smith (1966) and Cunia (1964) recommend weighting the dependent variable by the reciprocal of the variance where variance is unequal. An estimate of the variance can be obtained from the residuals squared.

The next step was to screen possible variables for correlation with residuals squared. The LOG of the residuals squared was screened against all possible combinations of the LOG's of the following variables:

1. Estimated cubic volume
2. Basal diameter minus minimum top diameter limit
3. Minimum top diameter limit
4. Height
5. Number of stems (juniper only).

The screen indicated the estimated cubic volume was the simplest, best overall predictor of residuals squared.

A LOG model of the following form was fitted to arrive at a preliminary estimate of variance as a function of estimated cubic volume:

$$LOG((\hat{Y} - Y)^2) = a + b\ LOG(\hat{Y})$$

where:

$(\hat{Y} - Y)^2$ = estimated variance, residuals squared
Y = field measured cubic volume
$\hat{Y}$ = estimated cubic volume from previous regression.

Substituting $\hat{V}$ for $(\hat{Y} - Y)^2$ and taking the antilog yielded:

$$\hat{V} = a\hat{Y}^b.$$

Therefore, the estimated weight was:

$$\hat{W} = \frac{1}{a\hat{Y}^b}.$$

Since a is a constant, it was dropped from the weight without affecting the relationship between the weights over estimated cubic volume. Dropping the constant yielded:

$$\hat{W} = \frac{1}{\hat{Y}^b}.$$

After the initial estimate of the weight was determined, the model was fitted using weighted least squares techniques. From the weighted model, a new estimate of variance and corresponding weight was obtained by again fitting a LOG model. Having obtained a new b value for the estimated weight, a new weighted regression was computed. The fourth iteration of this process yielded weights and regression coefficients essentially the same as the third iteration; this indicated the weights had stabilized and an appropriate weight had been found. The final weight used for the weighted regression was:

$$\hat{W} = \frac{1}{\hat{Y}^b}$$

where:

$\hat{Y}$ = estimated cubic volume from previous iteration
b = 1.43 for pinyon and 1.40 for juniper

Next another plot of the residuals was made over estimated cubic volume. The residuals were weighted by the square root of the weight used for the final weighted regression. The plot showed that the residuals:

1. Appeared in a horizontal band
2. Appeared balanced overall
3. Appeared unbalanced close to the origin
4. Appeared unbalanced for large estimated cubic volumes
5. For juniper only, had a conspicuous absence of positive residuals at 15 ft (0.42 m) offset by an absence of negative residuals at 25 ft (0.71 m).

Items 1 and 2 indicate appropriate weights were used. Items 3 and 4 indicate an intercept value should be included in the model. Item 5 indicates that two segments of the juniper model do not fit the data trends, but the lack of fit in one segment is offset by a corresponding lack of fit in another. Another examination of the means used to derive the original model revealed that to correct for the lack of fit in the two areas would result in an unrealistic model form. Therefore, the original model form was retained.

As a result of items 3 and 4, a new weighted regression was computed, this time with an intercept. The resulting intercepts were small [0.03 ft^3 (0.0008 m^3) for both pinyon and juniper], and the slope corrections were not unreasonable. A plot of the weighted residuals about the models with intercepts revealed that items 1 and 2 had been unchanged, items 3 and 4 had been corrected, and item 5 for juniper had been improved. Therefore, the weighted models with intercept values were accepted as the best linear unbiased estimators of cubic volume.

RESULTS

The final equations which follow have the following use restrictions:

1. The minimum top diameter limits must be in the range of 1 to 7 in (2.5 to 17.8 cm).

2. The number of stems cannot exceed 20.

3. The cubic volume predicted is total volume from ground line to point of minimum top diameter including bark and limbs.

4. The equations are considered representative of pinyon, Utah juniper, and Rocky Mountain juniper in northern New Mexico. Use elsewhere should be accompanied by suitable checks of applicability.

The pinyon cubic volume equation is:

$$V = c + bX^nH$$

where:

c (English units) = 0.02768

c (metric units) = 0.0007838

X (English units) = $D - TD$

X (metric units) = $\dfrac{D - TD}{2.54}$

D = basal diameter (at ground line) in inches for English units or centimeters for metric units

H = total tree height in feet for English units or meters for metric units

TD = minimum top diameter limit in inches for English units or centimeters for metric units

b (English units) = $0.08789 - 0.03675(11.0 - TD)^{0.35}$

b (metric units) = $0.0081652 - 0.0024637(27.94 - TD)^{0.35}$

n (English units) = $1.1 + 0.007(11.0 - TD)^{2.0}$

n (metric units) = $1.1 + 0.001085(27.94 - TD)^{2.0}$

V = gross cubic volume outside bark to the specified minimum top diameter limit including stump and limbs in cubic feet for English units or cubic meters for metric units.

Figure 1 illustrates the pinyon 1-inch top diameter volume surface showing the means used for model development plotted and connected to the surface by a vertical line. Note that more than 95 percent of the total data set was less than 14 in basal diameter.

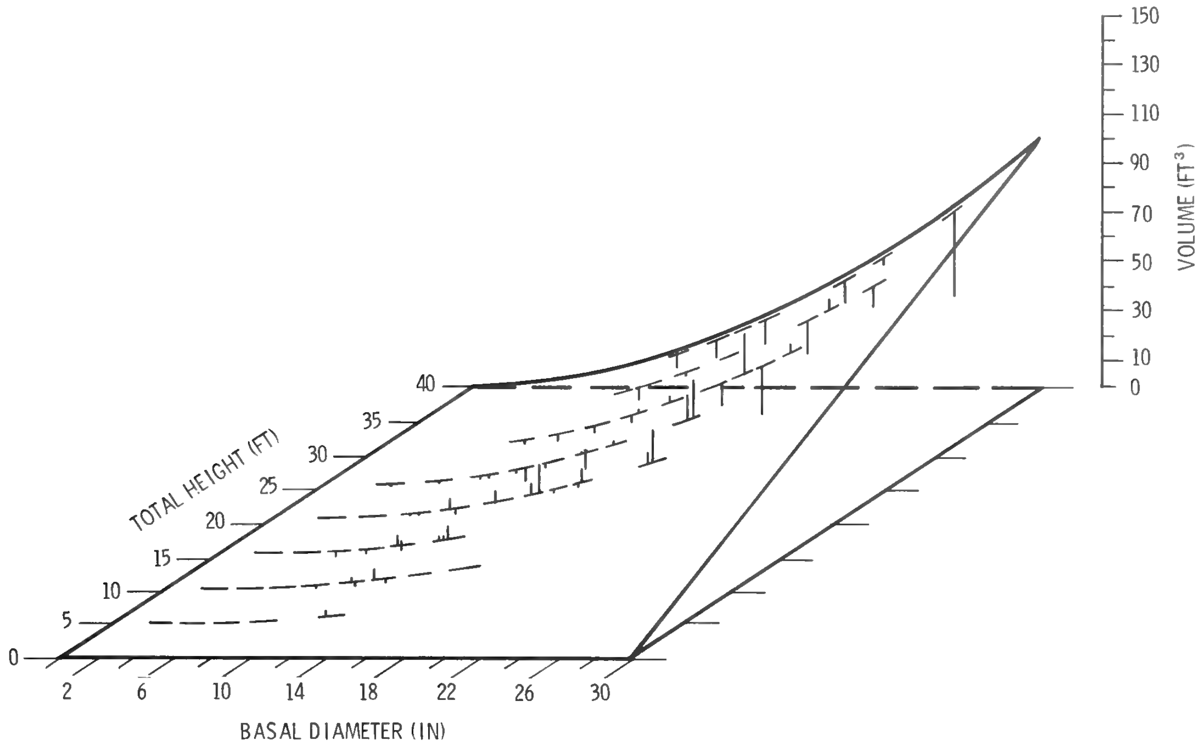

Figure 1.--Pinyon gross cubic-foot volume outside bark including stump and limbs to 1-inch minimum top diameter limit showing differences between observation means and corresponding predicted values.

The juniper cubic volume equation is

$$V = c + a^{n}bXH$$

where:

c (English units) = 0.03066

c (metric units) = 0.0008682

$$a = \frac{20.0 - \text{STEMS}}{19.0}$$

n (English units) = $2.25 + 0.38130(TD - 1.0)$

n (metric units) = $2.25 + 0.15012(TD - 2.54)$

X (English units) = $0.00491(D - TD)^{1.8} + 1.50147E\text{-}08(D - TD)^{5.0}$

X (metric units) = $0.0000852(D - TD)^{1.8} + 1.3194E\text{-}11(D - TD)^{5.0}$

STEMS = number of stems 3 in (7.62 cm) diameter and larger originating within first 12 in (0.3 m) above ground line. Trees less than 3 in (7.62 cm) basal diameter are considered single stemmed.

b (English units) = 1.08100 + 0.06263(TD - 1.0)

b (metric units) = 1.08100 + 0.0246575(TD - 2.54)

D = basal diameter (at ground line) in inches for English units or centimeters metric units

H = total tree height in feet for English units or meters for metric units

TD = minimum top diameter limit in inches for English units or centimeters for metric units

V = gross cubic volume outside bark to the specified minimum top diameter limit including stump and limbs in cubic feet for English units or cubic meters for metric units.

Figure 2 illustrates the juniper 1-in top diameter volume surface for single-stemmed trees showing the means used for model development plotted and connected to the surface by a vertical line. Note that more than 90 percent of the total data set was less than 14 inches basal diameter.

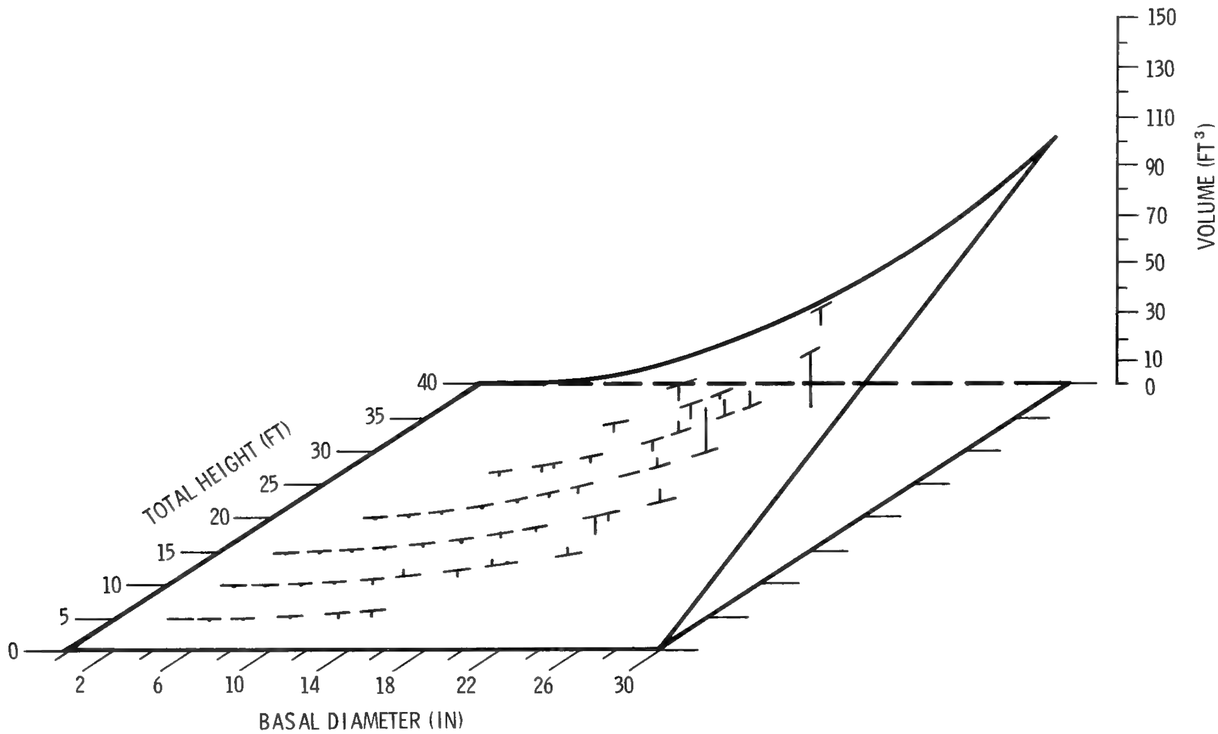

Figure 2.--Juniper gross-cubic volume outside bark including stump and limbs to 1-inch minimum top diameter limit for single stemmed trees showing differences between observation means and corresponding predicted values.

Figure 3 illustrates the top diameter effect for pinyon, and figure 4 illustrates the top diameter effect for single-stemmed juniper. Figure 5 illustrates the number-of-stems effect for juniper to a 1-in top diameter.

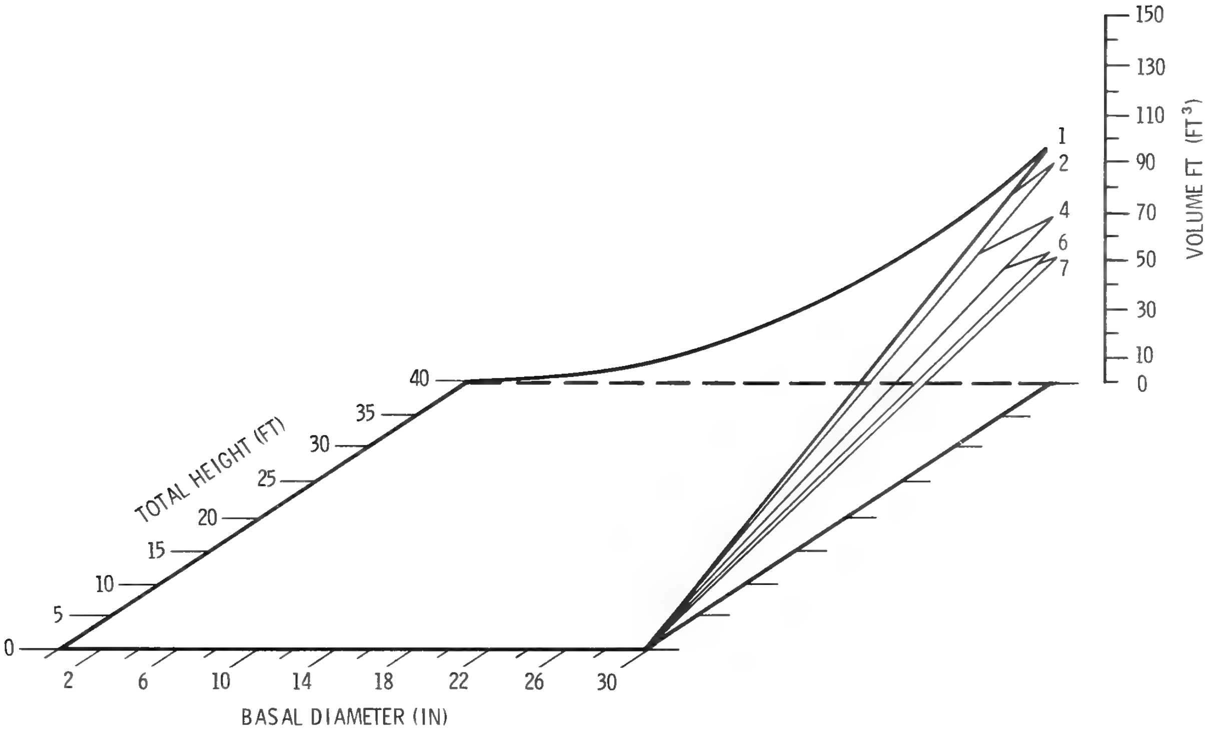

Figure 3.--Pinyon gross cubic-foot volume outside bark including stump and limbs to 1-inch, 2-inch, 4-inch, 6-inch, and 7-inch top diameter limits.

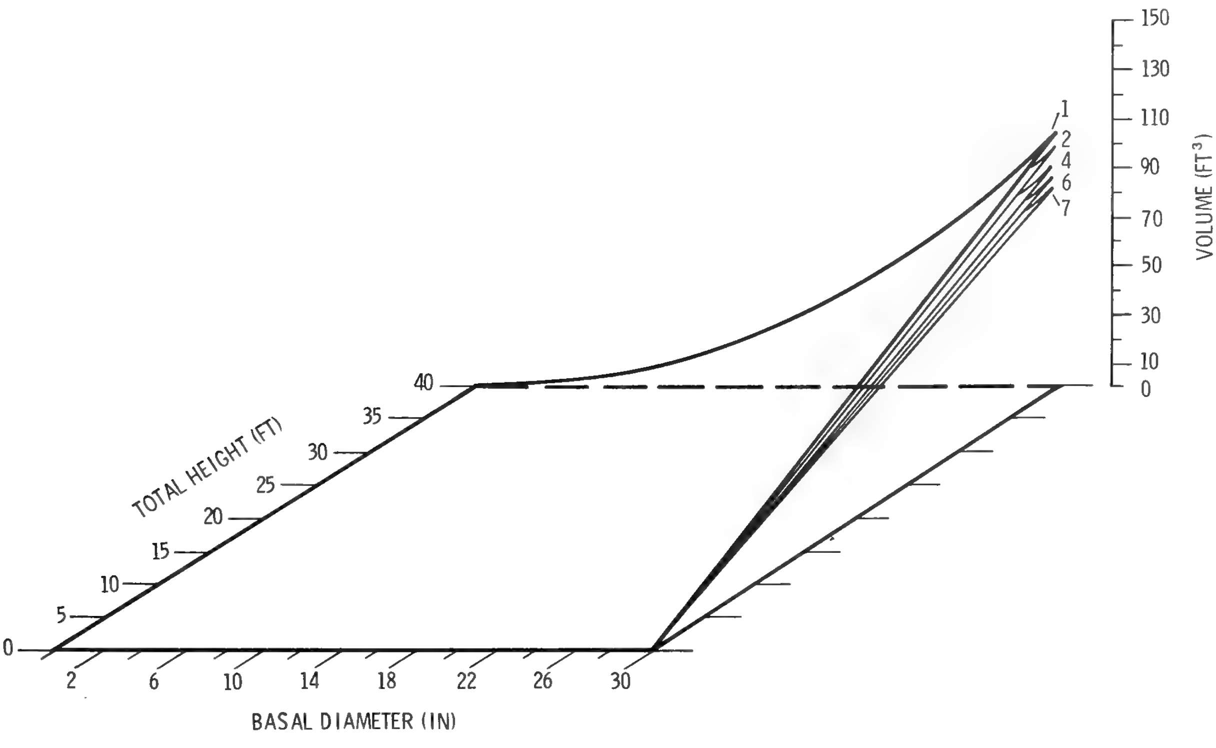

Figure 4.--Juniper gross cubic-foot volume outside bark including stump and limbs to 1-inch, 2-inch, 4-inch, and 7-inch minimum top diameter limit for single stemmed trees.

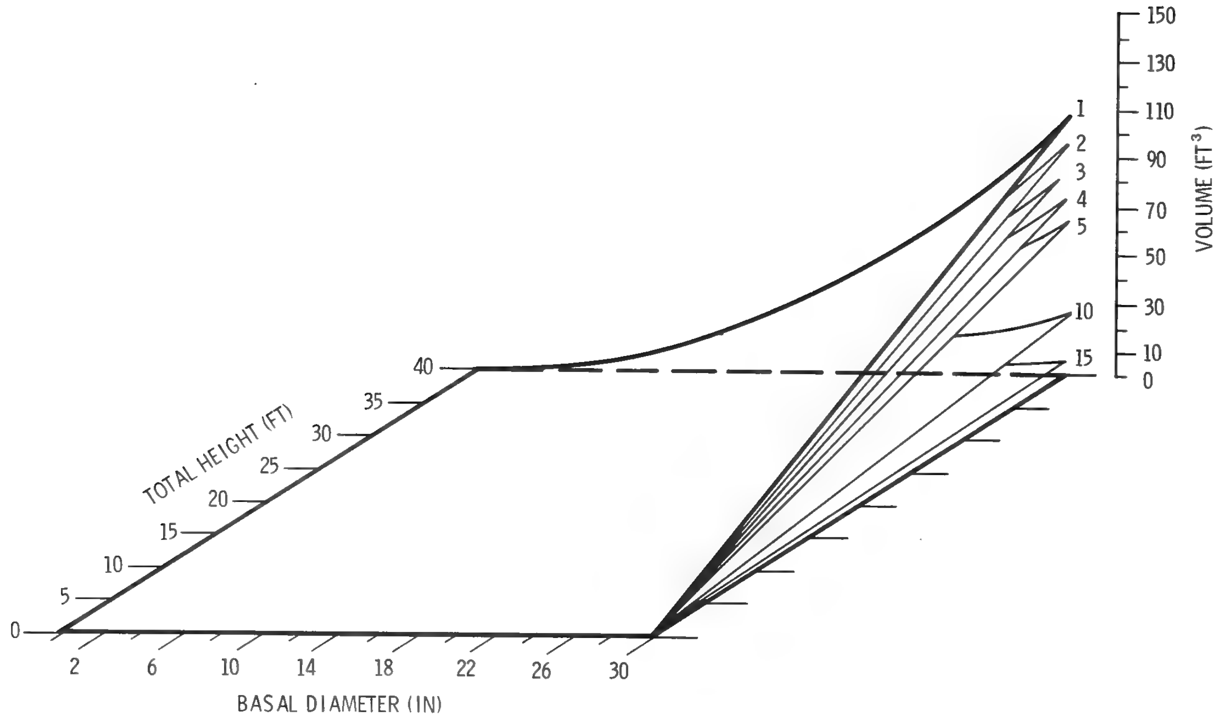

Figure 5.--Juniper gross cubic-foot volume outside bark including stump and limbs to 1-inch minimum top diameter for 1, 2, 3, 4, 5, 10, and 15 stemmed trees.

APPLICATION

The volume equations presented here are too complex to be solved easily with a desk calculator; therefore, two computer subroutines for computing individual tree volumes were written (English and metric units) and are included in Appendix I. The subroutines compute volumes in cubic feet with English unit input and cubic meters with metric unit input.

For desk calculator computation of tree volumes, tables 1 through 10 (in Appendix II) can be used. Tables 1 and 3 are in English units for pinyon and tables 5, 7, and 9 are in English units for juniper. Tables 2 and 4 are in metric units for pinyon and tables 6, 8, and 10 are in metric units for juniper.

To compute cubic-foot volume for a pinyon tree, first find the volume to a 1-in minimum top diameter for the tree in table 1. Then in table 3 find the appropriate proportion for the basal diameter of the tree and the desired minimum top diameter limit. Multiply the table 1 value by the table 3 value to obtain the cubic-foot volume of the tree to the desired minimum top diameter limit. Metric volume is obtained by the same procedure except that tables 2 and 4 are used instead of tables 1 and 3.

To compute cubic-foot volume for a juniper tree, first find the volume of a single-stemmed tree to a 1-in minimum top diameter in table 5. Second, in table 7 find the appropriate proportion for the basal diameter of the tree and the desired minimum top diameter limit. Finally, in table 9 find the appropriate proportion for the number of stems and the desired minimum top diameter limit. Multiply the values obtained from tables 5, 7, and 9 together to obtain the cubic-foot volume of the tree to the desired minimum top diameter limit. Metric volume is obtained by the same procedure except that tables 6, 8, and 10 are used instead of tables 5, 7, and 9.

Example: Assume a pinyon tree 10-in basal diameter and 30 ft tall. Determine the cubic-foot volume of this tree to a 4-in minimum top diameter limit. From table 1, the value 8.82 cubic feet is read. From table 3, the proportion 0.692 is read. The cubic-foot volume of this tree to a 4-in top is then 6.1 ft (8.82 X 0.692).

Example: Assume a juniper tree 12-in basal diameter, 20 ft tall, and having three stems originating within the first 12 in above ground line. Determine the cubic-foot volume of this tree to a 4-in minimum top diameter limit. From table 5,the value 8.034 is read. From table 7, the proportion 0.659 is read. From table 9, the proportion 0.686 is read. The cubic-foot volume of this tree to a 4-in top is then 3.63 ft (8.034 X 0.659 X 0.686).

CONCLUSIONS

The equations, computer subroutines, and tables presented in this paper are applicable throughout northern New Mexico for volume estimation in the pinyon-juniper type. Use outside of northern New Mexico should be accompanied by appropriate field checks.

PUBLICATIONS CITED

Barger, Roland L., and Peter F. Ffolliott.
1972. Physical characteristics and utilization of major woodland tree species in Arizona. USDA For. Serv. Res. Pap. RM-83, 80 p. Rocky Mtn. For. and Range Exp. Stn., Fort Collins, Colo.

Cunia, T.
1964. Weighted least squares method and construction of volume tables. For. Sci. 10(2):180-191.

Draper, N. R., and H. Smith.
1966. Applied regression analysis. 407 p. John Wiley and Sons, Inc., New York.

Grosenbaugh, L. R.
1967. REX--FORTRAN-4 SYSTEM for combinatorial screening or conventional analysis of multivariate regressions. USDA For. Serv. Res. Pap. PSW-44, 47 p. Pacific Southwest For. and Range Exp. Stn., Berkeley, Calif.

Howell, Joseph, Jr.
1940. Pinyon and juniper--a preliminary study of volume, growth, and yield. USDA Soil Conserv., Reg. 8, Bull. 71, For. Serv. 12, 90 p. Albuquerque, N. Mex.

Howell, Joseph, Jr.
1941. Pinyon and juniper woodlands of the Southwest. J. For. 39:542-545.

Howell, Joseph, Jr., and Bert R. Lexen.
1939. Fuelwood volume tables for Rocky Mountain red cedar (*Juniperus scopulorum* Sarg.). USDA For. Serv. Res. Note SW-68, 4 p. Southwest For. and Range Exp. Stn., Tucson, Ariz.

Jensen, Chester E.
1964. Algebraic description of forms in space. 57 p. USDA For. Serv., Cent. States For. Exp. Stn., Columbus, Ohio.

Jensen, Chester E.
1973. Matchacurve-3: Multiple-component and multidimensional mathematical models for natural resource studies. USDA For. Serv. Res. Pap. INT-146, 42 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

Jensen, Chester E.
1976. Matchacurve-4: Segmented mathematical descriptors for asymmetric curve forms. USDA For. Serv. Res. Pap. INT-182, 16 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

Jensen, Chester E., and Jack W. Homeyer.
1970. Matchacurve-1 for algebraic transforms to describe sigmoid- or bell-shaped curves. 22 p. USDA For. Serv., Intermt. For. and Range Exp. Stn., Ogden, Utah.

Jensen, Chester E., and Jack W. Homeyer.
1971. Matchacurve-2 for algebraic transforms to describe curves of the class X^n. USDA For. Serv. Res. Pap. INT-106, 39 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

Moessner, Karl E.
1962. Preliminary aerial volume tables for pinyon-juniper stands. USDA For. Serv. Res. Pap. INT-69, 12 p. Intermt. For. and Range Exp. Stn., Ogden, Utah.

APPENDIX I

```
      SUBROUTINE PJVOL(BD,TD,STEMS,HT,ISPEC,V)
C
C    *****
C    *  THIS SUBROUTINE COMPUTES GROSS CUBIC-FOOT VOLUME FOR INDIVIDUAL
C    *  PINYON AND JUNIPER TREES.
C         REQUIRED INPUTS --
C    *           1.  DIAMETER AT GROUND LINE
C    *           2.  MINIMUM TOP DIAMETER LIMIT
C    *           3.  NUMBER OF 3-INCH AND LARGER STEMS ORIGINATING WITHIN
C    *                FIRST 12 INCHES ABOVE GROUND LINE FOR JUNIPER ONLY
C    *           4.  TOTAL TREE HEIGHT
C    *           5.  SPECIES INDEX OF 1 FOR PINYON AND 2 FOR JUNIPER
C    *****
C
      D = BD - TD
      IF(TD.LT.1.0.OR.TD.GT.7.0.OR.STEMS.LT.0.0.AND.ISPEC.EQ.2.OR.STEMS.
     1 GT.20.0.AND.ISPEC.EQ.2.OR.HT.LT.0.0.OR.D.LT.0) GO TO 20
      IF(ISPEC.EQ.2) GO TO 10
      IF(ISPEC.NE.1) RETURN
C
C    *****
C    * PINYON
C    *****
C
      V = (.08789 - .03675 * (11. - TD)**.35) * (D**(1.1 + .007 * (11. -
     1 TD)**2.0)) * HT + .02768
      RETURN
C
C    *****
C    * JUNIPER
C    *****
C
   10 IF(STEMS.LE.1.0) STEMS = 1.000000
      V = (((20. - STEMS) / 19.)**(2.25 + .3813 * (TD - 1.)) * (1.08100 +
     1  .06263 * (TD - 1.)) * ((.00491 * D**1.8) + (1.50147E-08 * D
     2 ** 5.0))) * HT + .030664
      RETURN
   20 V = 0.0
      RETURN
      END
```

```
      SUBROUTINE PJVOL(BD,TD,STEMS,HT,ISPEC,V)
C
C    *****
C    *  THIS SUBROUTINE COMPUTES GROSS CUBIC-METER VOLUME FOR INDIVIDUAL
C    *  PINYON AND JUNIPER TREES.
C    *     REQUIRED INPUTS --
C    *         1.  DIAMETER AT GROUND LINE
C    *         2.  MINIMUM TOP DIAMETER LIMIT
C    *         3.  NUMBER OF 3-INCH AND LARGER STEMS ORIGINATING WITHIN
C    *              FIRST 12 INCHES ABOVE GROUND LINE FOR JUNIPER ONLY
C    *         4.  TOTAL TREE HEIGHT
C    *         5.  SPECIES INDEX OF 1 FOR PINYON AND 2 FOR JUNIPER
C    *****
C
      D = (BD - TD) / 2.54
      IF(ISPEC.EQ.2) D = D * 2.54
      IF(TD.LT.2.5.OR.TD.GT.15..OR.STEMS.LT.0.0.AND.ISPEC.EQ.2.OR.STEMS.
     1 GT.20.0.AND.ISPEC.EQ.2.OR.HT.LT.0.0.OR.D.LT.0) GO TO 20
      IF(ISPEC.EQ.2) GO TO 10
      IF(ISPEC.NE.1) RETURN
C
C    *****
C    * PINYON
C    *****
C
      V = (.0081652 - .0024637 * (27.94 - TD)**.35) * (D**(1.1 + .001085
     1 * (27.94 - TD)**2.0)) * HT + .0007838
      RETURN
C
C    *****
C    * JUNIPER
C    *****
C
   10 IF(STEMS.LE.1.0) STEMS = 1.000000
      V = (((20.0 - STEMS) / 19.0)**(2.25 +.15012 * (TD - 2.54)) * (1.0
     181 + .0246575 * (TD - 2.54)) * ((8.52E-05 * D**1.8) + (1.3194E-11
     2 * D**5.0))) * HT + .0008682
      RETURN
   20 V = 0.0
      RETURN
      END
```

APPENDIX II

Table 1.--*Pinyon gross cubic-foot volume outside bark including stump and limbs to 1-inch minimum top diameter limit*

Basal diameter (inches)	Total height (feet)									
	5	10	15	20	25	30	35	40	45	50
1	0	0	0	0	0	0	0	0	0	0
2	[1]0.056	0.084	0.112	0.140	0.168	0.196	0.224	0.252	0.280	0.309
4	.231	.433	.636	.839	1.042	1.245	1.448	1.651	1.854	2.057
6	.537	1.045	1.554	2.063	2.572	3.081	3.590	4.099	4.608	5.117
8	.960	1.893	2.825	3.758	4.690	5.623	6.555	7.488	8.420	9.353
10	1.494	2.960	4.425	5.891	7.357	8.823	10.289	11.755	13.221	14.687
12	2.131	4.235	6.339	8.442	10.546	12.650	14.753	16.857	18.961	21.065
14	2.869	5.711	8.553	11.394	14.236	17.078	19.919	22.761	25.603	28.444
16	3.704	7.381	11.057	14.734	18.410	22.087	25.763	29.440	33.116	36.793
18	4.633	9.239	13.844	18.450	23.055	27.661	32.267	36.872	41.478	46.083
20	5.654	11.280	16.907	22.533	28.160	33.786	39.412	45.039	50.665	56.292
22	6.765	13.502	20.239	26.976	33.713	40.450	47.187	53.924	60.661	67.398
24	7.963	15.899	23.835	31.770	39.706	47.642	55.577	63.513	71.449	79.385
26	9.248	18.469	27.690	36.911	46.131	55.352	64.573	73.794	83.014	92.235
28	10.618	21.209	31.800	42.391	52.982	63.573	74.163	84.754	95.345	105.936
30	12.072	24.117	36.161	48.206	60.250	72.295	84.340	96.384	108.429	120.473
32	13.608	27.189	40.770	54.351	67.932	81.512	95.093	108.674	122.255	135.836
34	15.226	30.425	45.623	60.821	76.020	91.218	106.417	121.615	136.814	152.012
36	16.924	33.821	50.717	67.614	84.510	101.407	118.303	135.200	152.096	168.993
38	18.702	37.376	56.050	74.724	93.398	112.072	130.746	149.420	168.094	186.768
40	20.558	41.088	61.618	82.148	102.678	123.208	143.738	164.268	184.798	205.328

[1]Blocked in area represents extent of data.

Table 2.--*Pinyon gross cubic-meter volume outside bark including stump and limbs to 2.5-centimeter minimum top diameter limit*

Basal diameter (centimeters)	Total height (meters) 1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0	13.5
2.5	0	0	0	0	0	0	0	0	0
5.0	0.0015	0.0023	0.0030	0.0038	0.0046	0.0053	0.0061	0.0068	0.0076
10.0	.0062	.0117	.0172	.0226	.0281	.0336	.0390	.0445	.0500
15.0	.0145	.0282	.0420	.0557	.0694	.0831	.0968	.1106	.1243
20.0	.0259	.0511	.0763	.1014	.1266	.1518	.1769	.2021	.2273
25.0	.0404	.0799	.1195	.1591	.1987	.2383	.2779	.3174	.3570
30.0	.0576	.1144	.1713	.2281	.2849	.3417	.3986	.4554	.5122
35.0	.0776	.1544	.2312	.3079	.3847	.4615	.5383	.6151	.6919
40.0	.1002	.1996	.2989	.3983	.4977	.5971	.6965	.7958	.8952
45.0	.1253	.2498	.3744	.4989	.6234	.7480	.8725	.9970	1.1216
50.0	.1530	.3051	.4573	.6095	.7616	.9138	1.0660	1.2182	1.3703
55.0	.1830	.3653	.5475	.7298	.9120	1.0943	1.2765	1.4588	1.6410
60.0	.2155	.4302	.6449	.8597	1.0744	1.2891	1.5038	1.7185	1.9332
65.0	.2503	.4999	.7494	.9989	1.2485	1.4980	1.7475	1.9971	2.2466
70.0	.2874	.5741	.8608	1.1474	1.4341	1.7207	2.0074	2.2941	2.5807
75.0	.3268	.6529	.9790	1.3050	1.6311	1.9571	2.2832	2.6093	2.9353
80.0	.3685	.7362	1.1039	1.4716	1.8393	2.2070	2.5747	2.9424	3.3101
85.0	.4123	.8239	1.2355	1.6470	2.0586	2.4701	2.8817	3.2932	3.7048
90.0	.4584	.9160	1.3736	1.8312	2.2888	2.7464	3.2040	3.6616	4.1192
95.0	.5066	1.0124	1.5182	2.0240	2.5298	3.0356	3.5414	4.0472	4.5530
100.0	.5569	1.1131	1.6692	2.2253	2.7815	3.3376	3.8937	4.4499	5.0060

Table 3.--*Pinyon gross cubic-foot volume proportion of 1-inch minimum top diameter limit*

Basal diameter (inches)	Minimum top diameter limit (inches)						
	1	2	3	4	5	6	7
2	1.000	0	0	0	0	0	0
4	1.000	0.677	0.300	0	0	0	0
6	1.000	.852	.637	0.411	0.192	0	0
8	1.000	.914	.765	.607	.454	0.305	0.154
10	1.000	.939	.819	.692	.574	.467	.365
12	1.000	.949	.842	.730	.630	.545	.472
14	1.000	.952	.850	.745	.655	.582	.525
16	1.000	.952	.851	.750	.664	.598	.550
18	1.000	.949	.847	.747	.665	.602	.560
20	1.000	.945	.842	.742	.660	.600	.561
22	1.000	.941	.835	.734	.653	.594	.557
24	1.000	.936	.828	.726	.644	.586	.551
26	1.000	.932	.820	.717	.635	.577	.542
28	1.000	.927	.813	.707	.625	.567	.533
30	1.000	.922	.805	.698	.615	.557	.523
32	1.000	.918	.797	.689	.605	.547	.513
34	1.000	.913	.790	.680	.596	.538	.504
36	1.000	.909	.783	.672	.587	.528	.494
38	1.000	.904	.776	.663	.577	.519	.485
40	1.000	.900	.769	.655	.569	.510	.476

Table 4.--*Pinyon gross cubic-meter volume proportion of 2.5 centimeter minimum top diameter limit*

Basal diameter (centimeters)	Minimum top diameter limit (centimeters)			
	2.5	5.0	10.0	15.0
2.5	1.00000	0	0	0
5.0	1.00000	0	0	0
10.0	1.00000	0.67600	0	0
15.0	1.00000	.85278	0.41110	0
20.0	1.00000	.91509	.60845	0.30488
25.0	1.00000	.94118	.69524	.46835
30.0	1.00000	.95158	.73448	.54803
35.0	1.00000	.95505	.75083	.58607
40.0	1.00000	.95465	.75559	.60266
45.0	1.00000	.95230	.75392	.60744
50.0	1.00000	.94892	.74867	.60563
55.0	1.00000	.94479	.74144	.60006
60.0	1.00000	.94036	.73314	.59228
65.0	1.00000	.93577	.72425	.58333
70.0	1.00000	.93118	.71519	.57372
75.0	1.00000	.92655	.70609	.56389
80.0	1.00000	.92203	.69711	.55406
85.0	1.00000	.91757	.68830	.54432
90.0	1.00000	.91324	.67974	.53481
95.0	1.00000	.90901	.67143	.52557
100.0	1.00000	.90489	.66340	.51662

Table 5.--*Juniper gross cubic-foot volume outside bark including stump and limbs to 1-inch minimum top diameter limit for single stemmed trees*

Basal diameter (inches)	Total height (feet) 5	10	15	20	25	30	35	40	45	50
1	0	0	0	0	0	0	0	0	0	0
2	0.057	0.084	0.110	0.137	0.163	0.190	0.216	0.243	0.270	0.296
4	.222	.414	.606	.798	.989	1.181	1.373	1.565	1.756	1.948
6	.512	.993	1.474	1.955	2.436	2.917	3.398	3.880	4.361	4.842
8	.913	1.796	2.678	3.561	4.443	5.326	6.208	7.091	7.973	8.856
10	1.421	2.811	4.201	5.591	6.981	8.371	9.761	11.151	12.541	13.931
12	2.032	4.033	6.033	8.034	10.035	12.036	14.037	16.038	18.039	20.040
14	2.746	5.461	8.177	10.892	13.607	16.323	19.038	21.753	24.469	27.184
16	3.566	7.102	10.638	14.174	17.709	21.245	24.781	28.316	31.852	35.388
18	4.498	8.965	13.432	17.899	22.367	26.834	31.301	35.768	40.235	44.703
20	5.548	11.066	16.583	22.101	27.618	33.136	38.653	44.171	49.689	55.206
22	6.728	13.426	20.123	26.821	33.518	40.216	46.913	53.611	60.308	67.006
24	8.052	16.073	24.094	32.115	40.136	48.157	56.178	64.199	72.220	80.241
26	9.536	19.042	28.547	38.053	47.558	57.064	66.570	76.075	85.581	95.086
28	11.203	22.375	33.547	44.719	55.891	67.063	78.236	89.408	100.580	111.752
30	13.077	26.122	39.168	52.214	65.260	78.306	91.352	104.398	117.444	130.490
32	15.187	30.343	45.500	60.656	75.813	90.969	106.125	121.282	136.438	151.595
34	17.568	35.106	52.643	70.181	87.719	105.256	122.794	140.331	157.869	175.407
36	20.259	40.488	60.716	80.945	101.173	121.402	141.630	161.859	182.087	202.316
38	23.304	46.577	69.851	93.124	116.397	139.671	162.944	186.217	209.491	232.764
40	26.752	53.474	80.196	106.918	133.639	160.361	187.083	213.804	240.526	267.248

Table 6.--*Juniper gross cubic-meter volume outside bark including stump and limbs to 2.5-centimeter top diameter limit for single stemmed trees*

Basal diameter (centimeters)	Total height (meters)								
	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0	13.5
2.5	0	0	0	0	0	0	0	0	0
5.0	0.0016	0.0023	0.0030	0.0037	0.0045	0.0052	0.0059	0.0066	0.0073
10.0	.0061	.0112	.0164	.0216	.0268	.0320	.0372	.0424	.0476
15.0	.0139	.0269	.0399	.0529	.0660	.0790	.0920	.1050	.1180
20.0	.0247	.0486	.0725	.0964	.1203	.1442	.1680	.1919	.2158
25.0	.0385	.0761	.1137	.1513	.1889	.2265	.2641	.3018	.3394
30.0	.0550	.1091	.1633	.2174	.2715	.3257	.3798	.4339	.4881
35.0	.0743	.1478	.2212	.2946	.3681	.4415	.5150	.5884	.6619
40.0	.0965	.1921	.2877	.3833	.4789	.5745	.6701	.7657	.8613
45.0	.1216	.2423	.3631	.4838	.6046	.7253	.8461	.9668	1.0875
50.0	.1499	.2990	.4480	.5971	.7461	.8952	1.0442	1.1933	1.3423
55.0	.1817	.3625	.5433	.7241	.9049	1.0857	1.2665	1.4473	1.6282
60.0	.2172	.4336	.6500	.8664	1.0827	1.2991	1.5155	1.7318	1.9482
65.0	.2571	.5132	.7694	1.0256	1.2818	1.5380	1.7942	2.0503	2.3065
70.0	.3017	.6024	.9032	1.2040	1.5048	1.8056	2.1064	2.4072	2.7080
75.0	.3517	.7025	1.0533	1.4042	1.7550	2.1058	2.4567	2.8075	3.1583
80.0	.4079	.8150	1.2220	1.6291	2.0362	2.4432	2.8503	3.2573	3.6644
85.0	.4712	.9416	1.4119	1.8823	2.3526	2.8230	3.2934	3.7637	4.2341
90.0	.5426	1.0843	1.6260	2.1678	2.7095	3.2512	3.7929	4.3346	4.8764
95.0	.6232	1.2455	1.8677	2.4900	3.1123	3.7346	4.3569	4.9792	5.6015
100.0	.7142	1.4276	2.1409	2.8542	3.5676	4.2809	4.9943	5.7076	6.4209

Table 7.--*Juniper gross cubic-foot volume proportion of 1-inch minimum top diameter limit for single stemmed trees*

Basal diameter (inches)	Minimum top diameter limit (inches)						
	1	2	3	4	5	6	7
2	1.000	0	0	0	0	0	0
4	1.000	0.517	0.168	0	0	0	0
6	1.000	.710	.448	0.230	0.074	0	0
8	1.000	.802	.610	.430	.270	0.138	0.044
10	1.000	.855	.709	.565	.428	.300	.188
12	1.000	.890	.775	.659	.544	.432	.325
14	1.000	.914	.822	.728	.631	.534	.439
16	1.000	.931	.857	.779	.697	.614	.530
18	1.000	.944	.883	.818	.749	.677	.604
20	1.000	.954	.904	.848	.789	.728	.663
22	1.000	.962	.919	.872	.822	.768	.712
24	1.000	.968	.932	.891	.847	.800	.751
26	1.000	.973	.942	.906	.868	.826	.782
28	1.000	.977	.949	.919	.884	.847	.808
30	1.000	.980	.956	.928	.898	.864	.829
32	1.000	.982	.961	.936	.908	.878	.845
34	1.000	.984	.964	.942	.916	.888	.859
36	1.000	.985	.967	.946	.923	.897	.869
38	1.000	.987	.970	.950	.928	.904	.878
40	1.000	.988	.972	.953	.932	.909	.884

Table 8.--*Juniper gross cubic-meter volume proportion of 2.5 centimeter minimum top diameter limit for single stemmed trees*

Basal diameter (centimeters)	Minimum top diameter limit (centimeters)			
	2.5	5.0	10.0	15.0
2.5	1.00000	0	0	0
5.0	1.00000	0	0	0
10.0	1.00000	0.51891	0	0
15.0	1.00000	.70932	0.23051	0
20.0	1.00000	.80120	.42956	0.13809
25.0	1.00000	.85445	.56423	.29935
30.0	1.00000	.88916	.65827	.43044
35.0	1.00000	.91313	.72624	.53256
40.0	1.00000	.93069	.77743	.61245
45.0	1.00000	.94372	.81655	.67559
50.0	1.00000	.95381	.84713	.72599
55.0	1.00000	.96161	.87121	.76637
60.0	1.00000	.96777	.89041	.79899
65.0	1.00000	.97264	.90566	.82532
70.0	1.00000	.97644	.91787	.84657
75.0	1.00000	.97951	.92762	.86366
80.0	1.00000	.98193	.93538	.87742
85.0	1.00000	.98382	.94155	.88841
90.0	1.00000	.98534	.94648	.89720
95.0	1.00000	.98656	.95041	.90424
100.0	1.00000	.98756	.95359	.90992

Table 9.--*Juniper gross cubic-foot volume proportion of single stemmed trees for multiple stemmed trees*

Number of stems	Minimum top diameter limit (inches)						
	1	2	3	4	5	6	7
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	.885	.867	.850	.832	.815	.799	.782
3	.779	.746	.715	.686	.657	.630	.604
4	.679	.636	.596	.558	.523	.490	.458
5	.588	.537	.491	.448	.410	.374	.342
6	.503	.448	.399	.355	.316	.281	.250
7	.426	.368	.319	.276	.239	.207	.179
8	.356	.298	.250	.210	.176	.148	.124
9	.292	.237	.193	.156	.127	.103	.084
10	.236	.193	.145	.113	.089	.069	.054
11	.186	.140	.105	.079	.060	.045	.034
12	.143	.103	.074	.053	.038	.027	.020
13	.106	.072	.049	.034	.023	.016	.011
14	.075	.048	.031	.020	.013	.008	.005
15	.050	.030	.018	.011	.006	.004	.002
16	.030	.017	.009	.005	.003	.002	.001
17	.016	.008	.004	.002	.001	.000	.000
18	.006	.003	.001	.000	.000	.000	.000
19	.001	.000	.000	.000	.000	.000	.000
20	.000	.000	.000	.000	.000	.000	.000

Table 10.--*Juniper gross cubic meter volume proportion of single stemmed trees for multiple stemmed trees*

Number of stems	Minimum top diameter limit (centimeters)			
	2.5	5.0	10.0	15.0
1	1.00000	1.00000	1.00000	1.00000
2	.88576	.86797	.83346	.80033
3	.77916	.74730	.68745	.63242
4	.68008	.63760	.56045	.49265
5	.58839	.53845	.45093	.37765
6	.50403	.44946	.35741	.28424
7	.42682	.37018	.27846	.20948
8	.35667	.30019	.21266	.15067
9	.29343	.23904	.15865	.10532
10	.23696	.18626	.11511	.07116
11	.18709	.14138	.08075	.04615
12	.14367	.10388	.05434	.02846
13	.10651	.07326	.03470	.01648
14	.07540	.04897	.02070	.00880
15	.05013	.03043	.01126	.00423
16	.03044	.01702	.00538	.00178
17	.01602	.00808	.00213	.00065
18	.00653	.00288	.00065	.00024
19	.00149	.00058	.00019	.00015
20	.00014	.00014	.00014	.00015

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Clendenen, Gary W.

1979. Gross cubic-volume equations...for pinyon and juniper trees in northern New Mexico. USDA For. Serv. Res. Pap. INT-228, 21 p. Intermt. For. and Range Exp. Stn., Ogden, Utah 84401.

Presents cubic-volume equations and tables for estimating gross cubic volume outside bark of individual pinyon and juniper trees in northern New Mexico; also shows procedures used in building mathematical model.

KEYWORDS: gross cubic-volume, equations, pinyon, juniper, mathematical models.

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- Boise, Idaho
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